Handout 4: The Equation

Philosophy 691: Conditionals
Northern Illinois University ⋆ Fall 2011
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§19. Other approaches

1. "The horseshoe analysis having failed, we must look further." Three other approaches to indicatives:

    (a) STRAWSON. $A \rightarrow C$ is true iff $A \supset C$ because of a connection between A and C. "Unsurprisingly (?), nobody has worked hard on trying to turn this 'connection' idea into a semantic analysis of indicative conditionals.".

    (b) POSSIBLE WORLDS. $A \rightarrow C$ is true iff at the possible world(s) in Φ where A is true, C is true. This was Stalnaker's idea, subsequently developed in a different way by Davis. Bennett distinguishes between the possible worlds approach and Lycan's analysis, but I think they belong to the same family. The central question for a possible worlds approach is: which worlds are in Φ; i.e., which possible A-worlds are the ones where C must be true for $A \rightarrow C$ to be true?

        The standard approach to counterfactuals is formally isomorphic. It says that $A \;\Box\!\!\rightarrow\; C$ is true iff at the 'closest' possible world(s) where A is true, C is true. A large body of work exists attempting to specify which worlds are the 'closest' in the relevant sense, and though controversies remain, there is something approaching a consensus answer to this question among theorists of counterfactuals. By contrast, the possible worlds account of indicatives has not been pursued with much vigor.

    (c) CONDITIONAL PROBABILITY. Central idea: the acceptability of $A \rightarrow C$ for a person is "governed by" her subjective conditional probability for C given A. This is the approach Bennett favors, and he will spend this chapter and the next five developing it.

2. Bennett will now shift from talking about assertibility to *acceptability*. "If I do not think that C is probable given A, I ought not even to *think* $A \rightarrow C$ let alone assert it."

3. Here is a big idea expressed without much discussion: "acceptability depends on probability and nothing else: you ought not to accept what you do not find probable, and there is no obstacle to your accepting what you do find probable" (p. 48). I have no evidence whatsoever that my magic 8-ball is reliable, and plenty of evidence that it's not. I ask it if my lottery ticket is a winner, and it replies "It is decidedly so." On this basis alone I become certain that I will win the lottery. According to Bennett there is "no obstacle to my accepting" that I will win. But obviously I have no evidence, warrant, or justification for accepting that I will win. So "acceptability" here has no direct connection to evidence, warrant, or justification.

4. The operative assumption henceforth, to be made precise at the end of the chapter: the acceptability of $A \rightarrow C$ to you is governed by your value for $P(C|A)$.

§20–23. Probability and the ratio formula

1. We have already discussed most of the material in these sections. A few comments:

2. Bennett initially uses the terms "absolute" and "relative" to distinguish between what are usually called "unconditional" and "conditional" probabilities, though in §21 he switches from "relative" to "conditional".

3. Most of §21 covers what we went over in the last class. Bennett's view of the role of probability theory:

> A probability *logic* is normative: it sets constraints on how people's degrees of closeness to certainty should behave and combine, as do the laws of ordinary logic and arithmetic. When Dummkopf grasps that being a cannibal entails being a carnivore, he *ought* to stop being more sure of Smith's being a cannibal than he is of his being carnivorous. This is the spirit in which we must understand all the axioms and theorems (p. 51).

But (you may ask) since probability logic only deals with probability functions that assign precise values to propositions, doesn't this presuppose that people's credences have precise numerical values? (Clarification: everyone will agree that people typically can't accurately *express* their credences by giving them precise values. The controversial presupposition is rather that their credences *have* precise values.) And if they don't, what are we to make of the claim that probability logic is normative? Bennett thinks at least some theorems would still have normative force: "The Addition Theorem, for instance, passes judgment on anyone who is almost certain of P while giving low probabilities to $P \,\&\, Q$ and $P \,\&\, \neg Q$."

Bennett doesn't mention this, but formally-oriented folks have a standard way of dealing with this issue. They represent a subject's credences not as *a* probability function, but as a *set* of probability functions, each of which is a permissible representation of her credences, but none of which gets privileged as *the* representation.

Obviously, this complicates the mathematics. Apart from that, it leads to challenges that always emerge when we try to represent an imprecise phenomenon using a precise language. Since for any set $\mathbf{P}$ of probability functions, a function P is either a member of $\mathbf{P}$ or it isn't, this approach presupposes that for every P there is an answer to the question: is P a permissible representation of S's credences? If S's credences are imprecise enough not to be accurately represented by a single probability function, it seems that they will also be imprecise enough that there will no precise boundary between permissible and impermissible representations of them.

4. Bennett uses $\pi(X|Y)$ to denote the conditional probability of X given Y, which I have been denoting using $P(X|Y)$. This typographical difference helps guard against mistakenly thinking that a probability function P has things of the form $X|Y$ or 'X given Y' in its domain. But it's too late for me to change my habits: just don't make that mistake.

5. Bennett gives what I called on the last handout the *conditionalization definition* of $P(\,|\,)$. But he endorses the ratio formula, denying only that it defines conditional probability. His discussion of why this approach is better is, to my mind, seriously baffling (p. 53):

   (a) He says that the ratio is not "always one's route to" the corresponding conditional probability; to "arrive at a value for $P(A \,\&\, C)$ one often has to proceed *through* a value for $P(C|A)$ or for $P(A|C)$, which means that we cannot use the right-hand side of the Ratio Formula to calculate the left," and that $P(C|A)$ "can often not be estimated by finding her values for $P(A \,\&\, C)$ and $P(A)$ and dividing one by the other." This all seems correct, at least assuming our credences have numerical values. But there are two ways of interpreting what it means:

   (b) First way: you often *know* a subject's $P(C|A)$ without *knowing* her $P(A)$ or $P(A \,\&\, C)$, and you sometimes learn the latter by plugging the former into the ratio formula. I agree, but don't see how this observation favors the ratio formula over the ratio definition. The definition of momentum (p) is given by the formula $p = mv$, but you can know an object's momentum without knowing its velocity (v) or mass (m).

   (c) Second way: often a subject *has* a value for $P(C|A)$ without *having* one for $P(A)$ or $P(A \,\&\, C)$. That Bennett thinks this is suggested by his approving citation of authors who have given "convincing cases" where a subject "[has] a value for $P(C|A)$ but none for $P(C \,\&\, A)$." Now, I think that this does occur. But wouldn't any such instance falsify the ratio formula? "In this particular case, $n/m = 0.7$, though n has no value." ???

   (d) My hypothesis about what's going on. Let your conditional credence in C given A be the value you get for C when you conditionalize on A. Bennett wants both:

      i. You can have a precise conditional credence in C given A without having credences in C or A.

      ii. Probability logic is normative for your conditional credences.

   Given (i), our conditional credences don't satisfy the ratio definition. But given (ii), we need to be able to treat our conditional credences as conditional probabilities, and hence as satisfying the ratio definition. I don't think the way to do this is to accept the ratio formula as a fact about our actual conditional credences. We should instead treat conditional probabilities as an idealization of our conditional credences.

6. Bennett discusses Hájek's "fierce and lavish" argument against the ratio definition ("What Conditional Probability Could Not Be," *Synthese* 137, 2003). The third objection Bennett reports is relevant to the issue just discussed. He describes the objection thus: when $P(A)$ and $P(C)$ are vague, then $P(C \,\&\, A)/P(A)$ is vague. But there are cases where someone has a precise value for $P(C|A)$ and only vague values for $P(A)$ and $P(C)$. In response, Bennett notes that in some of Hájek's examples A entails C, and he appeals to his proscription against independent conditionals to exclude these. He then says that he intends also to block conditionals where A and C are inconsistent (apparently these are relevantly like independent conditionals, and so can be ruled out if the latter can), thus allowing him to ignore Hájek's other examples of this phenomenon. So his response *seems* to indicate that he views the possibility of such cases as a threat to his use of the ratio formula, and seeks to exclude such cases on other grounds.

   But these exclusions won't do the job. I know that the garbage is picked up every other Tuesday. Let:

   T : *Today's Tuesday*

   G : *Today's a garbage day*

   Suppose my credences for both T and G are vague. Still, knowing what I do, I get a value of precisely $1/2$ for G when I conditionalize on T. So ignoring cases where A entails or is inconsistent with C will not enable Bennett to ignore all cases where a subject has vague credences in A and C but a precise conditional credence for C given A.

   Yet if it's okay to have cases where a subject has a precise value for $P(C|A)$ but *no* value for $P(A)$ or $P(C)$ (as Bennett seems to think), then surely it's not a problem for a subject to have *vague* values for $P(A)$ and $P(C)$ but a precise value for $P(C|A)$. Why does Bennett feel the need to respond to Hájek's objection by excluding the relevant cases? That's a real, not a rhetorical, question.

§23. INDICATIVE CONDITIONALS ARE ZERO-INTOLERANT

1. When $P(A) = 0$, the ratio formula has "no use".

2. "Rather than being a flaw in the formula, this result sheds light on its left-hand side, bringing into view an important consequence of the fact that indicative conditionals are devices for intellectually managing states of partial information, and for preparing the advent of beliefs that one does not currently have" (pp. 54-55).

3. The claim, boldly and categorically stated on page 55:

   ZERO-INTOLERANCE. Nobody ever has any use for $A \to C$ when for him $P(A) = 0$.

4. Three families of exceptions to the bold and categorical claim:

   (a) *Conversational stretches.*
   When you're certain A is false and are trying to convince someone else to agree, you can sometimes use $A \to C$ to get the job done. Look at the Spain example on p. 55. Here is the justification for the exception: "I base $A \to C$ on the Ramsey procedure as applied not to my system of beliefs but to someone else's."

   This could explain why $A \to C$ is *assertable* by someone for whom $P(A) = 0$. But does it explain why the conditional is *acceptable* to the speaker? If not, is that a problem?

   (b) *Conditionals inferred from conditionals.*
   $P(A) > 0$ in "If anybody admires the Rolling Stones, then he or she will never be Pope," and that implies "If a member of the Roman Curia admires the Rolling Stones, then he will never be Pope," in which $P(A) = 0$, but (I guess) the second conditional may still have some use. That's because the basis for accepting the second conditional is not its passing the Ramsey Test but its being a clear consequence of one that does.

   (This is a weird example: is the first "conditional" really a conditional? Doesn't it just say: nobody who admires the Rolling Stones will ever be Pope? For that matter, doesn't the second conditional just say: no member of the Roman Curia who admires the Rolling Stones will ever be Pope? Is there a better example?)

   Bennett's justification: "The thesis that indicative conditionals are zero-intolerant should be confined to ones that stand on their own feet." That sounds good but what does this mean? If any conditionals "stand on their own feet" I would think that independent conditionals do, but those are about to be excluded.

(c) Cases with "special features which enable one to go from A directly to C without having to consider how the rest of one's beliefs should be adjusted to make room for $P(A) = 1$" (p. 56):

   i. *Independents.*
   "[R]ather than having to accommodate $P(A) = 1$ [note: the text says '$P(A) = 0$' here but that can't be right] by adjusting your other probabilities, you can just dump it in there and go straight to $P(C) = 1$" (p. 57). Bennett doesn't give any examples of zero-tolerant independent indicatives.

   ii. *Non-interference conditionals.*
   Sometimes you are certain of C, and A's being true wouldn't interfere with your certainty in C. "Even if I didn't go to Spain last year, Gibralter still faces the Mediterranean." Umm, doesn't seeing whether A's truth would interfere with one's certainty in C involve "consider[ing] how the rest of one's beliefs should be adjusted to make room for $P(A) = 1$"?

   iii. *Conditionals implied by non-interference conditionals.*
   "The person accepts $A \to B$ as a non-interference conditional, where B asserts the existence of some item having a certain meaning or content, and C assigns to B a relational property ('is true', 'does not refer to anything', etc.) which it must have if A is true." Hmm, this is starting to sound pretty tortured... The example is a (very committed!) theist for whom P("God does not exist") $= 0$ and accepts "If God does not exist, then the first sentence of Genesis is false" on the basis of the non-interferer "Even if God does not exist, the first sentence of Genesis still says that God created the heavens and the earth".

5. A general-purpose rejoinder to purported counterexamples: is the conditional actually a subjunctive / counterfactual in disguise? 'Cuz *those* aren't zero-intolerant! "If I put my hand on the stove, I will get burned": don't you actually mean "If I were to put my hand on the stove, I would get burned"?

6. I am utterly certain that I have hands, and I accept that if I don't have hands, then I am having an extremely coherent hallucination. Exercise: respond on behalf of Bennett.

§24. THE EQUATION

1. How does your $P(C|A)$ govern the acceptability for you of $A \to C$? By this equation's being true:

   THE EQUATION. $P(A \to C) = P(C|A) = \frac{P(A \,\&\, C)}{P(A)}$ (provided $P(A) > 0$)

2. Old idea, circa mid-to-late 60's: the equation tells us indirectly what the truth-conditions of $A \to C$ are. They are whatever they need to be to make the equation true. Stalnaker's possible worlds analysis was originally intended to make those truth-conditions explicit; he wanted to give an "ontological analogue" of the equation.

3. This 60's optimism was pretty much put to rest by the mid-to-late 70's "triviality proofs" of Lewis (and many subsequent others). The short story on the triviality proofs is that they show that there can't be a two-place operator $\to$ for which the equation is true. (As usual, the short story is somewhat misleading. A slightly righter version of the short story: the triviality proofs show that if a probability function's domain is closed under $\to$, and the equation is true, then the function is "trivial" in various ways. For example, in such a function, for all X and Y, if $P(X \,\&\, Y) > 0$ and $P(X \,\&\, \neg Y) > 0$, then $P(Y|X) = P(Y)$.)

4. But all of the "triviality proofs" assume that $X \to Y$ is a proposition; i.e., that indicative conditionals are in the domain of a probability function. Give up this assumption, Bennett says, and we can hold on to the equation. And this is what he does: indicative conditionals "are not ordinary propositions that are, except when vagueness or ambiguity infects them, always either true or false" (59). In chapters 6 and 7 he'll give other, "recognizably philosophical" motivations for this "no-truth-value" view. But first, in chapter 5, the triviality proofs.

5. Goodness, isn't this a puzzling conjunction of claims?

   (a) No acceptable probability function P has a value for $P(A \to C)$.
   (b) The equation is true; i.e., $P(A \to C) = P(C|A)$ when $P(A) > 0$.

   Bennett appears to want both. (There may be ways to resist the triviality results without denying that $A \to C$ expresses something with truth-conditions. But Bennett seems not to be interested in pursuing that route.)