Handout 9: Stalnaker, Density, Limits

Philosophy 691: Conditionals
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Geoff Pynn

Stalnaker's version of the worlds theory

1. Stalnaker's theory makes use of a *selection function* s which maps proposition-world pairs to worlds.

   $A \mathrel{\Box\!\!\rightarrow} C$ is true at α iff C is true at $s\langle A, \alpha\rangle$.

   The selection function is constrained in the following ways:

   1. If $s\langle A, w\rangle = w'$, then A is true at w'.
   2. If A is true at w, then $s\langle A, w\rangle = w$.
   3. If A is true at $s\langle B, w\rangle$ and B is true at $s\langle A, w\rangle$, then $s\langle B, w\rangle = s\langle A, w\rangle$.

   The first constraint ensures that the selection function always picks a world where the antecedent is true. The second and third constraints enable us to treat the selection function as imposing a similarity ordering on worlds. Constraint 2 reflects the idea that no world is more similar to w than w is to itself: if A is true at w, then the most similar world to w where A is true is w itself. To see the motivation for Constraint 3 takes a bit more work. We want for $s\langle A, w\rangle$ to be the most similar world to w where A is true. Now suppose that for some B, $s\langle B, w\rangle$ is more similar to w than is $s\langle A, w\rangle$. Is A true at $s\langle B, w\rangle$? If the answer is yes, then $s\langle A, w\rangle$ isn't the world most similar to w where A is true after all. Constraint 3 ensures that the answer is no. (You may find it interesting to attempt to prove this for yourself.)

2. Interpreting the selection function in terms of similarity, Stalnaker's version of the worlds theory says:

   $A \mathrel{\Box\!\!\rightarrow} C$ is true at α iff at the A-world most similar to α, C is true.

3. Stalnaker's theory presupposes what Lewis calls *Stalnaker's Assumption*:

   STALNAKER. For any A, w, there is exactly one A-world most similar to w.

   Stalnaker's Assumption can be false in two ways. It may be that there are some $\langle A, w\rangle$ pairs such that for any A-world similar to w, another A-world is more similar to w. And it may be that for some $\langle A, w\rangle$ pairs, multiple A-worlds are equally similar to w. Thus Stalnaker's Assumption implies two weaker ones (ignore impossible A's):

   LIMIT. For every $\langle A, w\rangle$, at least one A-world w' is such that no A-world is more similar to w than w'.

   UNIQUENESS. For every $\langle A, w\rangle$, at most one A-world w' is such that no A-world is more similar to w than w'.

   Uniqueness is reflected in the fact that the selection function's domain consists of individual worlds. Limit is reflected in the fact that when $A \mathrel{\Box\!\!\rightarrow} C$ is true at w, $s\langle A, w\rangle$ is defined.

4. Note that neither assumption is a metaphysical claim about the nature of similarity or the relations that exist among possible worlds; it would be rash to suppose that for any respect of similarity, the limit or uniqueness assumptions hold. They should instead be understood as claims about the respects of similarity *relevant to the truth-conditions of a counterfactual.* When making (entertaining, evaluating, etc.) counterfactual judgments, we employ similarity measures that obey the limit and uniqueness assumptions. (Bennett makes this point on p. 180, see also Stalnaker (1978, p. 97), where he says that the selection function is undefined for an antecedent in a context that makes salient a limit-assumption-violating similarity measure.)

§70. THE LIMIT ASSUMPTION

1. On the last handout I tacitly made the limit assumption when I said that $A \mathrel{\Box\!\!\rightarrow} C$ is true at α iff:

   ... C is true at the "closest A-world(s)"

   ... $A \supset C$ is true at all worlds on or within the "smallest sphere around α containing A-worlds"

   ... etc.

   Lewis rejected the limit assumption: for him, $A \mathrel{\Box\!\!\rightarrow} C$ can be true at α even when for every A-world close to α, there is another A-world closer still. So the version of the worlds account I gave no the last handout is not quite his. Recall his "Ptolemaic astronomy" of worlds and note that there may be infinitely many spheres around α. Let 'A-sphere' = 'sphere containing at least one A-world'. Lewis's official theory says that $A \mathrel{\Box\!\!\rightarrow} C$ is true at α iff:

   (a) There is no A-sphere around α, or

   (b) There is an A-sphere around α such that $A \supset C$ is true at every world in that sphere.

   (a) concerns the vacuous case (i.e., when A is not possibly true). Case (b) can obtain even when the limit assumption fails: there can be an A-sphere around α even if there is no smallest A-sphere around α. This occurs when there are infinitely many A-spheres between α and an arbitrary A-sphere. In such a case, Lewis says that the counterfactual $A \mathrel{\Box\!\!\rightarrow} C$ expresses something like: "...as we take smaller and smaller [A-spheres] without end, eventually we come to ones in which C holds at every A-world" (*Counterfactuals*, p. 21).

2. Lewis's illustrates the failure of the limit assumption with this example (*Counterfactuals*, p. 20). Suppose line L is one inch long:

   L ________________

   Now consider all the worlds where L is longer than it actually is. What Bennett calls the *density assumption* implies that for any n, there's a world where L is $1/n$ inch longer than it actually is. Assuming that the worlds grow more similar to the actual world the higher n gets, the density assumption is incompatible with the limit assumption.

3. There are consequences to denying the limit assumption. Take (L):

   (L) If L were longer than it is, it would be at least $1 + 1/n$ inches long.

   The density assumption implies that for no value of n is (L) true. The limit assumption implies that there is some value of n for which (L) is true. Lewis rejected the limit assumption in favor of the density assumption, accepting that no instance of (L) is true to avoid postulating atoms of dissimilarity. You could instead accept atoms of dissimilarity, and get a true instance of (L). Which horn should you choose? If the question strikes you as scholastic, you have my sympathies.

4. Are there more serious consequences for denying the limit assumption? Perhaps. Consider (B):

   (B) If my beard were shorter than it is, my wife would complain less than she does.

   Assume density; deny limit; then there is no closest world shorter-beard world; for any world where it is shorter by $1/n$ inches, there is a world where it is shorter by $1/(n + 1)$ inches. Is there a shorter-beard sphere around α with respect to which (shorter beard $\supset$ less complaining) is true? No. In the very close shorter-beard spheres that material conditional is false: she cannot perceive that my beard is any shorter than it is, and so complains no less than she actually does. Since any larger shorter-beard sphere contains these smaller spheres, the material conditional is false with respect to those spheres, too. So denying the limit assumption seems to imply that (B) as false. But (B) is clearly true.

5. But at a deeper level, the problem is not with the (rejection of) the limit assumption. Assume limit: there are closest shorter-beard worlds. How much shorter is my beard in those worlds? If the answer is "imperceptibly so" then (B) is false, for in those worlds my wife's complaints do not abate. Density implied that there are no closest shorter-beard worlds, rendering (B) false. Limit dodges this bullet, but it allows that the closest shorter-beard worlds are too much like the actual world to make (B) true. What has gone wrong?

6. Initial diagnosis: by treating worlds as more and more similar to α as my beard approaches its actual length, we are focusing on a respect of similarity irrelevant to (B). When assessing (B), we do not care about worlds where my beard's length is an angstrom or two shy of its actual length. From Stalnaker (quoted by Bennett, p. 179):

> The selection function may ignore respects of similarity which are not relevant to the context in which the conditional statement is made. Even if, according to some general notion of overall similarity, i is clearly more similar to the actual world than j, if the ways in which it is more similar are irrelevant [to our present purposes], then j may be as good a candidate for selection as i.

This is a very sensible proclamation. If we accept it, we can see that the limit vs. density dialectic is tangential to the semantics for $\Box\!\!\rightarrow$. Let Lewis have density as a matter of pure metaphysics: in some senses (perhaps in some fundamental sense) there are no shorter-beard worlds most similar to α. Still, there are plenty of selection functions that take us from α to a shorter-beard world, and which have every right to be treated as determining a similarity ordering. (B) is true just in case the selection function it makes salient returns a world where my wife complains less than she does.

§71. THE CONSEQUENT AS CONTEXT

1. The limit assumption is compatible with the minimalness assumption:

> MINIMALNESS. If F-ness is a matter of degree and x is not F, then it is true that "If x were F, it would be minimally F".

Bennett points out that Stalnaker (1968, 1978) seems to accept minimalness. Bennett rejects it. I do not find his counterexamples especially persuasive (see p. 180). However, (B) is close to a counterexample: it is true, but only if the closest shorter-beard worlds are not minimally-shorter-beard worlds. Is (B)'s truth incompatible with minimalness? While they are uncomfortable partners, they may be compatible. But consider the stronger:

> MINIMALNESS*. If F-ness is a matter of degree and x is not F, then it is true that "if x were F, C" only if at the most similar worlds where x is F, x is minimally F.

(B) is decisive evidence against minimalness*, as is Bennett's "This party would have been better if it had been able to spread out on a bigger beach." With those models in mind, it is easy to construct more. Whatever we think of minimalness, we must accept that minimal difference cannot be a requirement for maximal similarity.

2. So when can we skip over minimally different worlds when searching for the most similar world(s) to α to evaluate $A \Box\!\!\rightarrow C$? Bennett says the answer is underwritten by the following principle of charity:

> CHARITY. As far as you reasonably can, interpret what a speaker says in such a way that it is neither obviously false nor so obviously true as to be not worth saying (p. 181).

If you utter 'S' and S's content is underdetermined in some way, charity enjoins me to resolve the underdetermination in ways that favor your assertion's being true without trivializing it. We're amongst some fourth grade girls, one of whom, Sally, is just under five feet. You say, "Sally's tall." That sentence doesn't determine a precise content; I need to do some interpretive work to understand what you've said. Here are three attempts, each permitted by the general features of the sentence you've uttered:

T Sally's tall for a toddler.

R Sally's tall for a fourth grader.

F Sally's tall for an NBA center.

T is too easily true, and F is too clearly false. R is just right: it's neither so easily true to render your utterance pointless, nor so clearly false as to render it unacceptable. Given these choices, charity enjoins me to interpret you as meaning R. Note that the initial clause of charity is essential. There are circumstances in which it would be perverse to interpret "Sally's tall" as true, and the first clause cancels our charitable obligations in such circumstances.

3. Applied to $A \mathrel{\Box\!\!\rightarrow} C$, the principle directs us to "understand the antecedent" in accordance with our "estimate of what [the speaker] must mean if the entire conditional is to be neither trivial nor ridiculous" (p. 182). I wouldn't put it this way. I'd say that the principle implies that the estimate should constrain which selection functions are viable candidates (Stalnaker) or which similarity relation orders the worlds (Lewis). I'm not sure what "understanding the antecedent" has to do with it.

4. The consequent plays a key role in the application of the principle. Adopting Stalnaker's framework for expository purposes: our choice of a selection function in assessing an assertion of $A \mathrel{\Box\!\!\rightarrow} C$ is partly constrained by whether "In $s\langle A, \alpha\rangle$, C" will come out true. If it is trivially true or an obvious falsehood on s, then we should not choose s. Of course, as above, this general guideline is subject to the "as far as is reasonable" limitation. There are circumstances in which a speaker might have a reason for asserting a trivial counterfactual, or considering an obviously false one. And some speakers might unwittingly assert counterfactuals which could only be true with respect to a bizarre selection function. In cases like these, charity may reasonably be withheld.